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Assessment and Investment Model (AIM)

by

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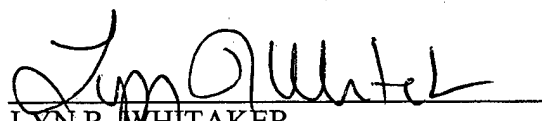Reproduction of all or part of this report is authorized.

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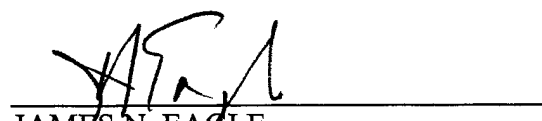Associate Professor of
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Abstract

The AIM model has been under development for two years, and currently takes the form of a Mixed-integer optimization model formulated in GAMS. This document summarizes current status and delineates possible future development opportunities.

Background and Status of AIM

In World War II, most air-to-ground munitions were simple, unguided gravity bombs. It was only near the end of that war when some tentative experiments and Japan's kamikaze fleet revealed the much-improved combat effectiveness of munitions that could detect and home in on a target. Even in Desert Storm, most of the munitions used were still MK-82 500-pound gravity bombs dropped by B-52s. Such bombs have their uses even today, but they are no longer the dominant method of conducting air-to-ground warfare by the United States.

The spectrum of munitions available to today's weaponeer is breathtaking in its scope, offering munitions that differ by orders of magnitude in cost, effectiveness, and potential for attrition to the launching platform. A Tomahawk may cost $1,000,000 while a gravity bomb of equivalent destructive power may cost only $1,000. There are many intermediate choices, and, even putting cost aside, a Tomahawk is not always the most effective weapon. Figure 1 shows some of the possibilities.
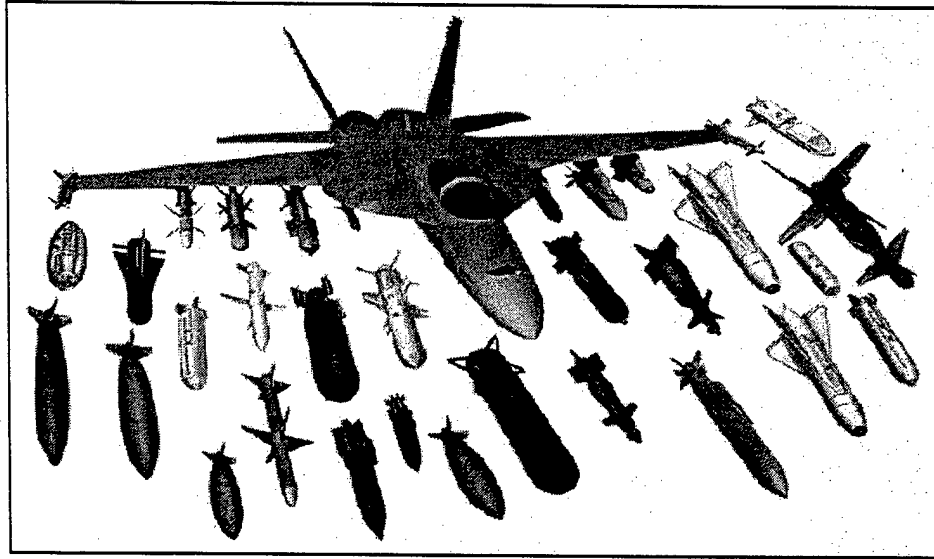
Figure 1: An F/A-18 with some of its available munitions

In these circumstances, deciding which weapons to stockpile for potential wars is a delicate tradeoff between money, combat effectiveness, and attrition to friendly aircraft. The weapon mix used in the next war will be so unlike the mix used in the last war that an appeal to history is unreliable. Billions of dollars are involved. How should all this money be spent? Answering that question in a scientific manner is the object of AIM.

At the moment, the Navy's munitions requirements are determined by the Non Nuclear Ordnance Requirements (NNOR) process [e.g., McGee-Pasceri, 2002]—a process that is not constrained by fiscal considerations. After the NNOR requirements are determined, there remains the problem of determining how to spend a given munitions budget. Given the natural tendency of a process unconstrained by budget considerations to emphasize expensive weapons, it is not surprising that the real budget is usually insufficient to support the entirety of NNOR, or even a large fraction of it. This leads to a quandary as to how AIM should be formulated:

1. Perhaps the best thing to do is to put a cost constraint into NNOR, so that munitions requirements are determined in the presence of pressure to minimize cost. This is, in a sense, the ideal solution, but it is not easy because it involves changing NNOR.

2. Or perhaps NNOR should be left as is, with the object of AIM being to somehow buy "as much of NNOR as possible" within the budget constraints.

The main charm of this approach is that it exploits, rather than replaces, the heavy investment that the Navy has already made in NNOR. Much less data is required.

We have tried both approaches, the first in 2002 and the second in 2003. The 2002 work led to formulation of the problem as a mathematical program that would replace the crucial combat requirement part of NNOR with an analytic battle model. The battle model would reflect the crucial tradeoffs between money, combat effectiveness, and attrition, so that the effects of low budgets could be seen directly in increased attrition and in targets not killed. Other parts of NNOR, the training requirements, for example, would be retained. A partial specification for that model (AIM1) is included here as Appendix 1. The economic part of AIM1 was never completely specified, sources for some of the data needed were never identified, and there still remained questions about exactly how much of NNOR would be retained. The projected difficulty of completing these tasks was partially responsible for the decision to switch to the second approach in 2003, which offered a better prospect of completion in a timely manner.

The second approach (AIM2) is documented in the Master's thesis of John Bruggeman [2003]. AIM2 is a completely specified mathematical program, with known sources for all of the data required. NALC has an operating copy of the software.

When confronted with a budget that is insufficient for a set of "requirements", some organizations simply enforce a 50% cut in all requirements, or whatever percentage is required for feasibility. AIM2 is capable of doing that, and (we feel) would be useful if it did nothing else. The reason for this claim is that AIM2 has an economic model that deals with maintaining a viable industrial base over multiple years in the planning horizon. This is a complicating economic feature that makes dealing with multi-year munitions requirements intuitively difficult. However, AIM2 does not merely enforce a fixed percentage cut on all munitions, but instead optimizes the "tier-level" achieved. The details of tiers are best read in Bruggeman's thesis, but the basic idea is to exploit the fact that NNOR does not simply announce the total number of munitions of each type that is required, but instead separates the total number into Combat Requirements (CR), Total Training Requirements (TTR), Current Operations/Force Protection Requirements

(CO/FPR), and Strategic Reserve Requirements (SRR). These four components deserve different priorities, depending on the munition involved. AIM2's tier-levels express this differing emphasis, and thus usefully generalize the idea of simply forcing all munitions to take the same percentage cut.

Development Alternatives

The original quandary is still present. Useful though AIM2 may be, it cannot illuminate the tradeoffs between money, combat effectiveness, and attrition, except though a tier level that is a surrogate for the latter two. The idea of attacking targets with appropriate regard for money and attrition is not present in NNOR, because NNOR is not cost constrained. AIM2 does not include the concepts of "attack" or "attrition", except by reflection from NNOR. Thus, there will always be good reasons for resuming work on AIM1. The economic part of the work on AIM2 would be transferable to AIM1, but making AIM1 into a useful model would still be a significant undertaking over a multi-year period.

Although a return to AIM1 may be tempting, the idea of replacing or competing with NNOR is sufficiently daunting that further development of AIM2 is probably the more attractive alternative for 2004. The best ways of improving AIM2 will no doubt emerge in the process of using it to determine munitions purchases, a process that the authors of this report are prepared to support. Here are some possibilities that are already known:

1. **Improved heuristics**. The current mixed-integer formulation turns out to be a difficult mathematical program to solve optimally. Bruggeman developed a few constructive heuristics to provide quick, approximately optimal solutions. Improvements include the addition of local search routines such as tabu search that attempt to improve the solution(s) found by the constructive heuristics. These would (essentially) swap procurement funds between munition types in a given year, and also swap funds between years. These search techniques can also be randomized in a fairly straightforward manner, allowing advanced heuristic approaches such as simulated annealing (with low implementation overhead, once basic local search is working) and genetic algorithms (with much more effort).

2. **Improved user controls**. AIM has several features that allow decision makers to control the structure of procurement decisions that it examines. The current set of features covers many important types of control, including persistent solutions, removing weapons from consideration, aggregate industrial-base constraints, etc. However, many of these can be tailored further to meet specific procurement requirements that may arise in practice. Useful controls can be elevated to the main interface page, and others can be removed. Solution techniques (whether optimal or heuristic) can be tailored to deal with these modifications appropriately.

3. **Improved interface.** The data input and output routines are automated, and this significantly improves the usability of AIM. However, standardizing the structure of the input data files and the various sections of the spreadsheet interface would help decision makers learn AIM quickly, and would help them explore a wider range of good procurement decisions quickly. This effort would be in direct collaboration with current users of the system.

4. **Floating tier levels.** In the current formulation, each munition has fixed thresholds for each tier level. For example, tier 2 might require 100 units, whereas tier 3 might require 400. Buying 300 units would still leave the tier level at 2, since 300 is not sufficient for tier 3. But there is no fundamental reason why the tier level concept has to be integer-valued, and allowing it to float would have some sensitivity advantages. In that case, buying 300 units would correspond to tier 2.67, since 300 is 2/3 of the way from 100 to 400. Implementing these floating tier levels would not be difficult.

5. **Economics**. AIM munitions are currently bought in discrete lots, rather than continuous quantities, with the costs and quantities of the various lots being inputs. For the most part, munitions become cheaper as more are bought in a given year; that is, except for the possibility of temporarily shutting down the production line, cost is a concave function of quantity up until the maximum production capacity is reached. This concavity could be exploited computationally if it were guaranteed, but the trouble is that there are currently some exceptions in the database. Although minor, they still prevent

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taking advantage of concavity. What should be done about this? Do they represent anything real? Should they simply be smoothed to be concave? A comparison with RADSS [Fusco, 2002], which models cost as a continuous, concave function of quantity, might prove instructive.

Summary

There are two versions of AIM. AIM2 is based on tier-levels and is currently operational. Several possibilities for the further development and enhancement of AIM2 have been identified. However, the NNOR/AIM2 process cannot deal quantitatively with tradeoffs between money, attrition, and combat effectiveness on account of its sequential nature: first the NNOR process determines munitions requirements without regard to money, and then AIM2 imposes budgetary constraints. Integrating the two processes would produce something like AIM1, as outlined in the Appendix 1.

Appendix (Formulation of AIM1)

Introduction

AIM1 was formulated in 2002, but never implemented. The formulation is recorded here for possible future reference, should it ever be felt desirable to embed an air-to-ground model within AIM.

AIM1 is an optimizing model intended to determine optimal values for p_{mn}, the best amount of munition m to buy in year n of a multi-year procurement program, for all values of m and n. Equivalently, the problem is to determine the best values of s_{mn}, the inventories of each munition type at the start of year n. With one major exception, AIM1 is a conventional inventory model where the current year's inventory s_{mn} is augmented by current purchases p_{mn} and decremented by the current year's usage in order to determine next year's inventory $s_{m,n+1}$. The current year's usage includes everything that is forecastable about munitions usage, including training.

The major exception is that AIM1 must face the possibility that a war will have to be fought, starting with whatever the current munitions inventories are at the beginning of the war. The war is assumed to be large, commensurate with Defense Planning Guidance, and, except for the time of its initiation, predictable. It is modeled in detail, including the possibility that some initial munitions inventories will be small enough to force some targets to be attacked by munitions or tactics that are not ideal. As an indirect result, friendly platforms may be lost or enemy targets may escape that might otherwise have been killed. The AIM1 objective function is composed of three penalty terms, one for the cost of munitions purchased before the war, one for attrition (friendly losses) in the war, and one for excess enemy survival in the war. These three phenomena are felt to be the most important ones that must be dealt with by any model that claims to give advice on "optimal" munitions purchases for the Department of the Navy (DoN).

Uncertain War

It would be analytically easiest to have a definite time at which the war starts. The trouble with this option is that it might lead to AIM1's producing solutions where purchases are minimal up until just before the war starts, and which then "ramp up" to

provide sufficient initial munitions for the war. In reality, the USA cannot reliably predict when or even if there will be a war, but must constantly be prepared for the possibility. AIM1 actually fights a virtual war in each year of the planning horizon in order to emulate this uncertainty, combining the results into a single term in the objective function. Many wars are planned, although at most one is fought. The analytic cost of this method of dealing with uncertainty is a proliferation of variables in what is called the Battle Submodel. It remains to be seen whether the method is computationally feasible with realistic datasets.

The war has so far been envisioned as an air-to-ground war. We realize that munitions such as mines and torpedoes also need to be planned for, but air-to-ground munitions are the most difficult because of the variety of ways in which ground targets can be attacked from the air.

The transition from peacetime to wartime is handled by introducing a completely different acquisition process. Regardless of initial inventories, munitions are produced at the rate P_{mt} (upper case symbols denote data) for munition m in time period t of the war (warfare is modeled on a finer scale than the yearly munitions procurement cycle, so time periods may be only a few days or weeks long). There is no cost term in the objective function associated with this production; that is, the principle is "peacetime cost, wartime effectiveness". The transition from war to peace is not modeled, so there can be at most one war within the planning horizon.

Analytic Formulation

AIM1 consists of a single mathematical program with a single objective function, but it is easiest to describe in terms of a Master Logistics model that manipulates peacetime munitions inventories s_{mn}, feeds the inventories to a Battle Submodel, and receives from the submodel the attrition and excess survival terms of the master objective function. The cost term of the objective function depends on the industrial base model, which would presumably resemble the model in AIM2. It will not be shown explicitly here.

Indexes:

i for aircraft type

j for partition (offboard targeting available, etc.)

k for target type

l for tactic

m for munition type

n for year of the planning horizon, the logistics resolution being one year

w for weather type

t for time period within a war

Data:

S_{it}≡sortie rate for type i (aggregate sorties per day) in period t

C_i=attrition cost of losing one aircraft of type i

V_k=value of one target of type k

F_{ijkwt}=fraction of type i sorties in partition j directed against targets of type k in weather w in time period t of a war ($\sum_{jkwt} F_{ijkwt} = 1$ for all i)

$\alpha_{ijklmwt}$≡attrition probability for sorties in period t

$Q_{ijklmwt}$≡excess miss probability, relative to the most effective (l,m)

N_{iklmt}≡number of munitions type m used per sortie type i using tactic l

D_t≡length of period t in a war, smaller than a year

I_m≡initial inventory of munition m at start of planning horizon

P_{mt}≡rate of providing munitions of type m in period t of a war

Y≡number of years in the planning horizon

P_n≡probability that a war starts in year n ($\sum_{n=1}^{Y} P_n \le 1$)

T_{mn}≡training requirements for munition m in year n

W≡number of periods in a war

Variables:

$x_{ijklmnwt}$≡sortie rate (sorties per day)

y_{mnt}≡rate of using type m (number per day)

p_{mn}≡munitions of type m purchased at the start of year n

s_{mn}≡supply of type m at start of year n, barring war

u_{mnt}≡supply of type m at start of period t in a war starting in year n

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r_{int}=rate at which aircraft type i are lost

q_{knt}=excess rate of missing targets of type k

Constraints:

The Master Logistics Model

$s_{m1}=I_m$

$$s_{m,n+1} = s_{mn} + p_{mn} - T_{mn};\ 1 \leq n \leq Y\text{-}1$$

{Budget constraints would be included in this master model. Much more needs to be said, but for the moment just let "cost(p_{mn})" be the cost of total munitions purchases over the entire time horizon.}

$$\text{objective1}=\text{cost}(p_{mn})+ \sum_{int} P_n C_i D_t r_{int} + \sum_{knt} P_n V_k D_t q_{knt}$$

{objective1 is an expected-value objective based on the input probabilities P_n. The obvious choice is to make all P_n equal, giving equal emphasis to every year. The second and third terms come from the Battle Submodel.}

$$\text{objective2}=\text{cost}(p_{mn})+\max_n \{ \sum_{it} C_i D_t r_{int} + \sum_{kt} V_k D_t q_{knt} \}$$

{objective 2 includes the worst-case war, which is likely to be the first year of the planning horizon. The input probabilities P_n are not needed.}

The Battle Submodel (if the battle begins in year n with initial inventories s_{mn})

$$\sum_{lm} x_{ijklmnwt} = S_{it}F_{ijkwt};\ \forall ijknwt$$

$$\sum_{jklmw} x_{ijklmnwt}\alpha_{ijklmwt} = r_{int};\ \forall int$$

$$\sum_{ijlmw} x_{ijklmnwt}Q_{ijklmwt} = q_{knt};\ \forall knt$$

$$\sum_{ijklw} x_{ijklmnwt}N_{iklmt} = y_{mnt};\ \forall mnt$$

$$u_{mnt} = s_{mn};\ t = 1;\ \forall mn$$

$$D_t y_{mnt} \leq u_{mnt};\ \forall mnt$$

$$u_{mn,t+1} = u_{mnt} + D_t(P_{mt} - y_{mnt});\ t = 1,...,W - 1;\ \forall m,n$$

In order, these constraints enforce that:

- The total number of sorties must be as the data specifies.
- The attrition rate coefficients determine attrition.

- The miss probabilities determine excess targets missed.

- The munitions use coefficients determine munitions usage.

- Initial munitions inventories shall be as specified by the Master Model.

- Munitions use in period t cannot exceed the stock at period start.

- Inventory balance equations over all periods of the war.

Further Comments

The user would get feedback of various kinds ("You better hope there is no war for the next three years!", or maybe a graph of total penalty costs versus year), in addition to aggregations of variables as desired.

Arguments can be made either way about which objective function more closely mimics the actual problem. The max function in the second can be implemented by introducing one additional variable, so there is no difference in computational difficulty.

All sorties that DoN is capable of generating must be used in the Battle Submodel, but one (l,m) pair consumes 0 munitions and has 0 attrition, so you can always use such null sorties it if you run out of real munitions. Null sorties do not have 0 for Q, however, so using them will make the number of targets missed get large.

There is no list of targets to be killed. The general idea is that DoN (as far as air-to-ground is concerned) is limited mainly by the size of its air force, and will surely be at the mercy of a joint Air Tasking Order (ATO) process that assigns targets commensurate with its capabilities. There will be no difficulty finding targets in a big war. With unlimited munitions and no concern for attrition, all of the q-variables would be 0. Otherwise, there will be some missed targets. Thus, the basic problem is that missed targets, money, and attrition cannot all be 0 at the same time, hence the need for tradeoffs and optimization. At the moment, there is no negative term in the objective function for munitions that survive the war. Perhaps there should be.

Inventories should really be kept in terms of munition components, rather than munitions, but making that change is conceptually simple. The Battle Submodel is linear as stated, and would remain linear if it were component-based. However, the Master Logistics model will likely need to include integer variables to handle the startup and shutdown of production lines.

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